Exercise 12.1 (Revised) - Chapter 12 - Surface Areas And Volumes - Ncert Solutions class 10 - Maths

Updated On 11-02-2025 By Lithanya

Chapter 12: Surface Areas and Volumes - NCERT Solutions Class 10 Maths

Unless stated otherwise, take $\pi = \frac{22}{7}$.

Ex 12.1 Question 1.

2 cubes each of volume $64 \mathrm{~cm}^3$ are joined end to end. Find the surface area of the resulting cuboid.

Answer.

Volume of cube $= (\text{ Side })^3$
According to question, $(\text{ Side })^3 = 64$
$\Rightarrow (\text{ Side })^3 = 4^3$
$\Rightarrow \text{ Side } = 4 \mathrm{~cm}$

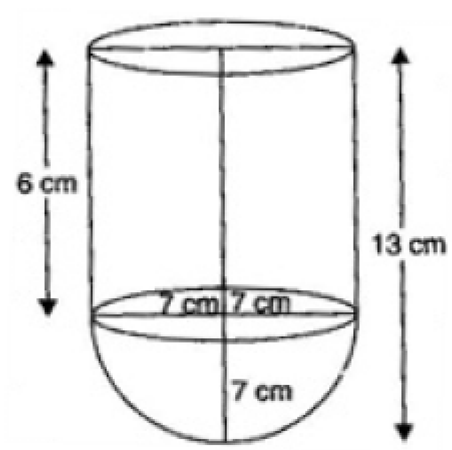
For the resulting cuboid, length $(l) = 4 + 4 = 8 \mathrm{~cm}$, breadth $(b) = 4 \mathrm{~cm}$ and height $(h) = 4 \mathrm{~cm}$
Surface area of resulting cuboid $= 2(lb + bh + hl)$
$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)$
$= 2(32 + 16 + 32)$
$= 2 \times 80 = 160 \mathrm{~cm}^2$

Ex 12.1 Question 2.

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is $14 \mathrm{~cm}$ and the total height of the vessel is $13 \mathrm{~cm}$. Find the inner surface area of the vessel.

Answer.

$\because$ Diameter of the hollow hemisphere $= 14 \mathrm{~cm}$
$\therefore$ Radius of the hollow hemisphere $= \frac{14}{2} = 7 \mathrm{~cm}$



Total height of the vessel $= 13 \mathrm{~cm}$
$\therefore$ Height of the hollow cylinder $= 13 - 7 = 6 \mathrm{~cm}$
$\therefore$ Inner surface area of the vessel
= Inner surface area of the hollow hemisphere + Inner surface area of the hollow cylinder
$= 2\pi(7)^2 + 2\pi(7)(6)$

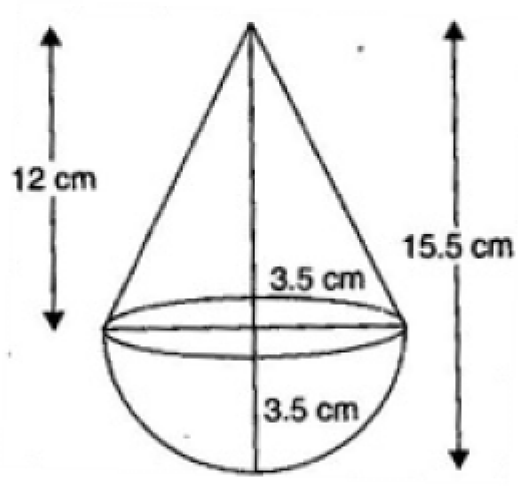
$= 98\pi + 84\pi = 182\pi$
$= 182 \times \frac{22}{7} = 26 \times 22 = 572 \text{ cm}^2$

Ex 12.1 Question 3.

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Answer.

Radius of the cone $= 3.5 \text{ cm}$
$\therefore$ Radius of the hemisphere $= 3.5 \text{ cm}$



Total height of the toy $= 15.5 \text{ cm}$
$\therefore$ Height of the cone $= 15.5 - 3.5 = 12 \text{ cm}$
Slant height of the cone $= \sqrt{(3.5)^2 + (12)^2}$
$= \sqrt{12.25 + 144}$
$= \sqrt{156.25} = 12.5 \text{ cm}$
$\therefore$ TSA of the toy $=$ CSA of hemisphere $+$ CSA of cone
$= 2\pi r^2 + \pi r l$
$= 2\pi(3.5)^2 + \pi(3.5)(12.5)$
$= 24.5\pi + 43.75\pi = 68.25\pi$
$= 68.25 \times \frac{22}{7} = 214.5 \text{ cm}^2$

Ex 12.1 Question 4.

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Answer.

Greatest diameter of the hemisphere $=$ Side of the cubical block $= 7 \text{ cm}$
$\therefore$ TSA of the solid = External surface area of the cubical block + CSA of hemisphere

$= \left\{6(7)^2 - \pi\left(\frac{7}{2}\right)^2\right\} + 2\pi\left(\frac{7}{2}\right)^2$
$=> \left(294 - \frac{49}{4}\pi\right) + \frac{49}{2}\pi$
$= 294 + \frac{49}{4}\pi$
$= 294 + \frac{49}{4} \times \frac{22}{7}$
$= 294 + \frac{77}{2}$
$= 294 + 38.5 = 332.5 \text{ cm}^2$

Ex 12.1 Question 5.

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer.

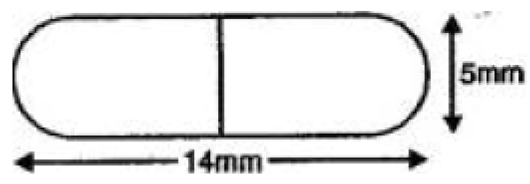
$\because$ Diameter of the hemisphere $= l$, therefore radius of the hemisphere $= \frac{l}{2}$
Also, length of the edge of the cube $= l$
$\therefore$ Surface area of the remaining solid $=$ total surface area of cubical block + curved surface area of hemispherical - area of circular base

$= 2\pi\left(\frac{l}{2}\right)^2 + 6l^2 - \pi\left(\frac{l}{2}\right)^2$
$= \pi\left(\frac{l}{2}\right)^2 + 6l^2$
$= \frac{\pi l^2}{4} + 6l^2$

$= \frac{1}{4}l^2(\pi + 24)$

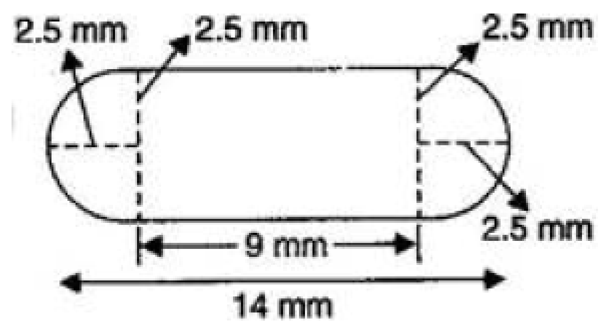
Ex 12.1 Question 6.

A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see figure). The length of the entire capsule is $14 \mathrm{~mm}$ and the diameter of the capsule is $5 \mathrm{~mm}$. Find its surface area.



Answer.

Radius of the hemisphere $= \frac{5}{2} \mathrm{mm}$
Let radius $= r = 2.5 \mathrm{~mm}$
Cylindrical height $=$ Total height - Diameter of sphere $= h = 14 - (2.5 + 2.5) = 9 \mathrm{~mm}$
Surface area of the capsule $=$ CSA of cylinder + curved Surface area of 2 hemispheres



$= 2\pi rh + 2\left(2\pi r^2\right)$
$= 2\pi\left(\frac{5}{2}\right)(9) + 2\left\{2\pi\left(\frac{5}{2}\right)^2\right\}$
$= 45\pi + 25\pi$
$= 70\pi = 70 \times \frac{22}{7} = 220 \mathrm{~mm}^2$

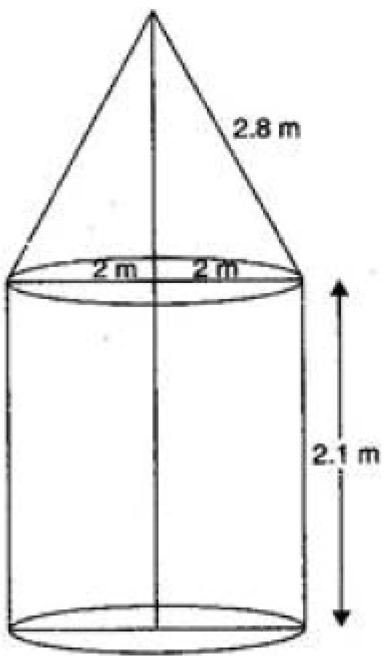
Ex 12.1 Question 7.

A tent is in the shape of a cylinder surmounted buy a conical top. If the height and diameter of the cylindrical part are $2.1 \mathrm{~m}$ and $4 \mathrm{~m}$ respectively and the slant height of the top is $2.8 \mathrm{~m}$, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs. 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

Answer.

Diameter of the cylindrical part $= 4 \mathrm{~cm}$

$\therefore$ Radius of the cylindrical part $= 2 \mathrm{~cm}$
TSA of the tent = CSA of the cylindrical part + CSA of conical top



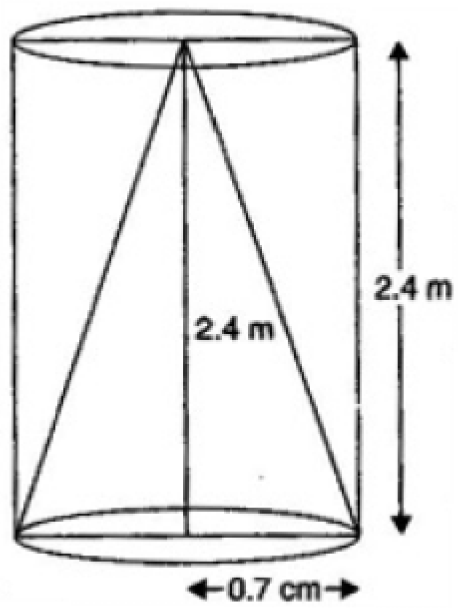
$= 2\pi(2)(2.1) + \pi(2)(2.8)$
$= 8.4\pi + 5.6\pi$
$= 14\pi$
$= 14 \times \frac{22}{7}$
$= 44 \mathrm{~m}^2$
$\therefore$ Cost of the canvas of the tent of $1 \mathrm{~m}^2 =$ Rs. 500
cost of canvas of the tent of $44 \mathrm{~m}^2 =$
$= 44 \times 500 =$ Rs. 22000

Ex 12.1 Question 8.

From a solid cylinder whose height is $2.4 \mathrm{~cm}$ and diameter $1.4 \mathrm{~cm}$, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Answer.

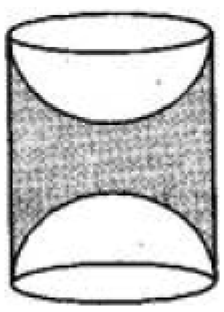
Diameter of the solid cylinder $= 1.4 \mathrm{~cm}$
$\therefore$ Radius of the solid cylinder $= 0.7 \mathrm{~cm}$
$\therefore$ Radius of the base of the conical cavity $= 0.7 \mathrm{~cm}$



Height of the solid cylinder $= 2.4 \mathrm{~cm}$
$\therefore$ Height of the conical cavity $= 2.4 \mathrm{~cm}$
$\therefore$ Slant height of the conical cavity $= \sqrt{(0.7)^2 + (2.4)^2}$
$= \sqrt{0.49 + 5.76}$
$= \sqrt{6.25} = 2.5 \mathrm{~cm}$
$\therefore$ TSA of remaining solid = curved surface area of cylinder + area of upper circular part + curved surface area of conical part
$= 2\pi(0.7)(2.4) + \pi(0.7)^2 + \pi(0.7)(2.5)$
$= 3.36\pi + 0.49\pi + 1.75\pi$
$= 5.6\pi$
$= 5.6 \times \frac{22}{7} = 17.6 \mathrm{~cm}^2$
$= 18 \mathrm{~cm}^2$ (to the nearest cm^2)

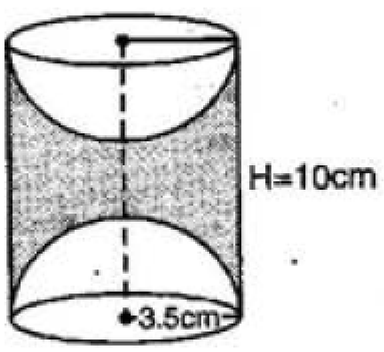
Ex 12.1 Question 9.

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder as shown in figure. If the height of the cylinder is $10 \mathrm{~cm}$ and its base is of radius $3.5 \mathrm{~cm}$, find the total surface area of the article.



Answer.

TSA of the article $= 2\pi r H + 2\left(2\pi r^2\right) =$ curved surface area of cylinder + curved surface area of 2 hemispheres
$= 2\pi(3.5)(10) + 2\left[2\pi(3.5)^2\right]$
$= 70\pi + 49\pi$
$= 119\pi$
$= 119 \times \frac{22}{7}$
$= 374 \mathrm{~cm}^2$



Chapter 12: Surface Areas and Volumes - NCERT Solutions Class 10 Maths

Unless stated otherwise, take $\pi=\frac{22}{7}$.

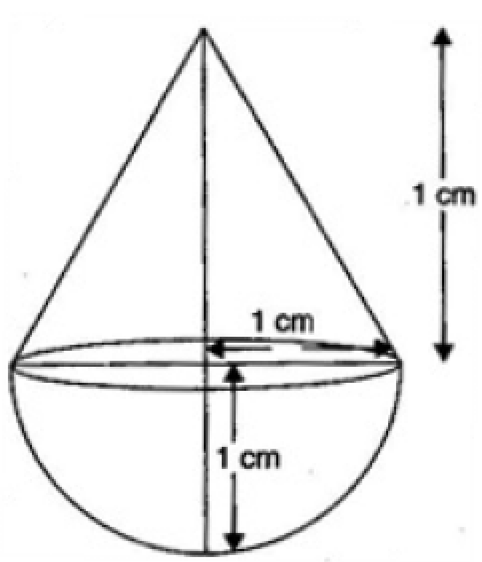
Ex 12.2 Question 1.

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to $1 \mathrm{~cm}$ and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Answer.

For hemisphere, Radius $(r)=1 \mathrm{~cm}$

$$\text { Volume }=\frac{2}{3} \pi r^3$$
$$=\frac{2}{3} \pi(1)^3$$
$$=\frac{2}{3} \pi \mathrm{cm}^3$$



$\text{For cone, Radius of the base } (r)=1 \mathrm{~cm}$

Height $(h)=1 \mathrm{~cm}$

$$\text { Volume }=\frac{1}{3} \pi r^2 h$$
$$=\frac{1}{3} \pi(1)^2 \times 1$$
$$=\frac{1}{3} \pi \mathrm{cm}^3$$

$\therefore$ Volume of the solid = Volume of hemisphere + Volume of cone

$=\frac{2}{3} \pi+\frac{1}{3} \pi=\pi \mathrm{cm}^3$

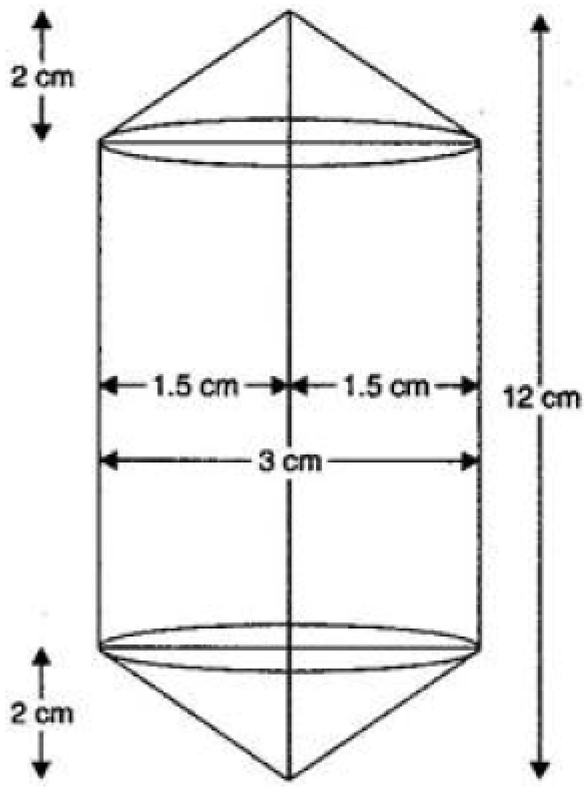
Ex 12.2 Question 2.

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is $3 \mathrm{~cm}$ and its length is $12 \mathrm{~cm}$. If each cone has a height of $2 \mathrm{~cm}$, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Answer.

For upper conical portion, Radius of the base $(r)=1.5 \mathrm{~cm}$

$\text{Height } \left(h_1\right)=2 \mathrm{~cm}$



$$\text{Volume } =\frac{1}{3} \pi r^2 h_1$$
$$=\frac{1}{3} \pi(1.5)^2 \times 2$$
$$=1.5 \pi \mathrm{cm}^3$$

For lower conical portion, Volume $=1.5 \pi \mathrm{cm}^3$

For central cylindrical portion:

Radius of the base $(r)=1.5 \mathrm{~cm}$

Height $\left(h_2\right)=12-(2+2)=8 \mathrm{~cm}$

Volume $=\pi r^2 h_2=\pi(1.5)^2 \times 8=18 \pi \mathrm{cm}^3$

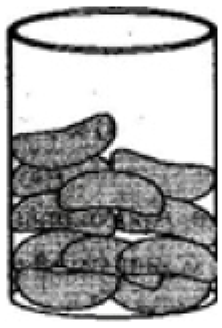
$\therefore$ Volume of the model $=1.5 \pi+1.5 \pi+18 \pi=$ volume of top cone + volume of bottom cone + volume of cylindrical part

$=21 \pi$

$=21 \times \frac{22}{7}=66 \mathrm{~cm}^3$

Ex 12.2 Question 3.

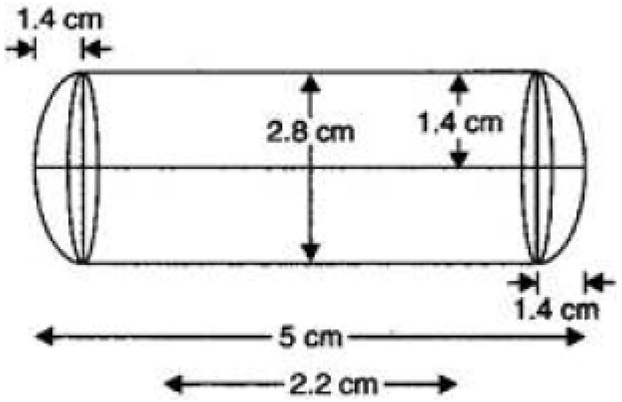
A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends, with length $5 \mathrm{~cm}$ and diameter $2.8 \mathrm{~cm}$ (see figure).



Answer.

Volume of a gulab jamun $=\frac{2}{3} \pi r^3+\pi r^2 h+\frac{2}{3} \pi r^3=$ volume of 2hemisphere + volume of cylinder

$=\frac{2}{3} \pi(1.4)^3+\pi(1.4)^2 \times 2.2+\frac{2}{3} \pi(1.4)^3$



$= \frac{4}{3}\pi(1.4)^3 + \pi(1.4)^2 \times 2.2$

$= \pi(1.4)^2\left[\frac{4 \times 1.4}{3} + 2.2\right]$

$= \pi \times 1.96\left[\frac{5.6+6.6}{3}\right] = \frac{1.96 \times 12.2}{3}\pi \text{cm}^3$

$\therefore$ Volume of 45 gulab jamuns

$= 45 \times \frac{1.96 \times 12.2}{3}\pi$

$= 15 \times 1.96 \times 12.2 \times \frac{22}{7}$

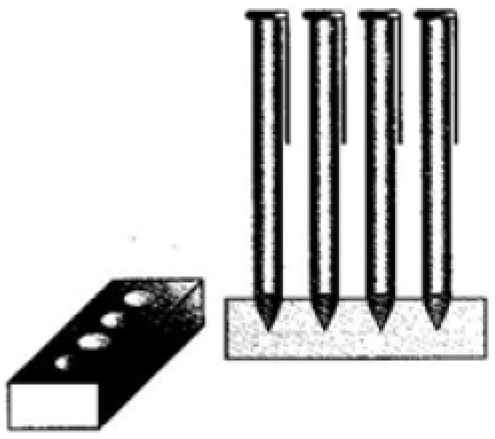
$= 1127.28 \text{ cm}^3$

$\therefore$ Volume of syrup $= 1127.28 \times \frac{30}{100} = 30\%$ of volume of 45 gulab jamun

$= 338.184 \text{ cm}^3 = 338 \text{ cm}^3$ (approx.)

Ex 12.2 Question 4.

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth id 1.4 cm. Find the volume of wood in the entire stand (see figure).



Answer:

For Cuboid:

$l = 15 \text{ cm}$

$b = 10 \text{ cm}$

$\text{h} = 3.5 \text{ cm}$

Volume of the cuboid $= l \times b \times h$

$= 15 \times 10 \times 3.5$

$= 525 \text{ cm}^3$

For Cone: $\text{r} = 0.5 \text{ cm}$

$\text{h} = 1.4 \text{ cm}$

Volume of conical depression $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

$= \frac{11}{30} \text{cm}^3$

$\therefore$ Volume of four conical depressions $= 4 \times \frac{11}{30} = 1.47 \text{ cm}^3$

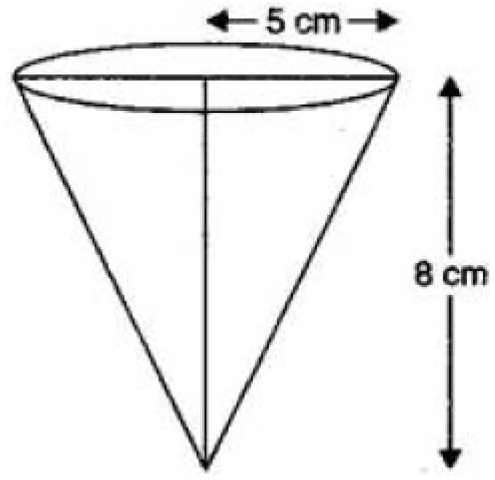
$\therefore$ Volume of the wood in the entire stand = volume of cuboid - volume of 4 conical depression $= 525 - 1.47 = 523.53 \text{ cm}^3$

Ex 12.2 Question 5.

A vessel is in the form of inverted cone. Its height is 8 cm and the radius of the top, which is open, is 5 **cm**. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer.

For cone, Radius of the top $(r) = 5 \text{ cm}$ and height $(h) = 8 \text{ cm}$



Volume of cone $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(5)^2 \times 8$
$= \frac{200}{3}\pi \mathrm{cm}^3$

For spherical lead shot, Radius $(R) = 0.5 \mathrm{~cm}$
Volume of spherical lead shot $= \frac{4}{3}\pi R^3$
$= \frac{4}{3}\pi(0.5)^3$
$= \frac{\pi}{6}\mathrm{cm}^3$

Volume of water that flows out $= \frac{1}{4}$Volume of the cone
$= \frac{1}{4} \times \frac{200\pi}{3} = \frac{50\pi}{3}\mathrm{cm}^3$

Let the number of lead shots dropped in the vessel be n.
$n*$ volume of spherical shot = volume of water flows out

$\therefore n \times \frac{\pi}{6} = \frac{50\pi}{3}$
$\Rightarrow n = \frac{50\pi}{3} \times \frac{6}{\pi}$
$\Rightarrow n = 100$

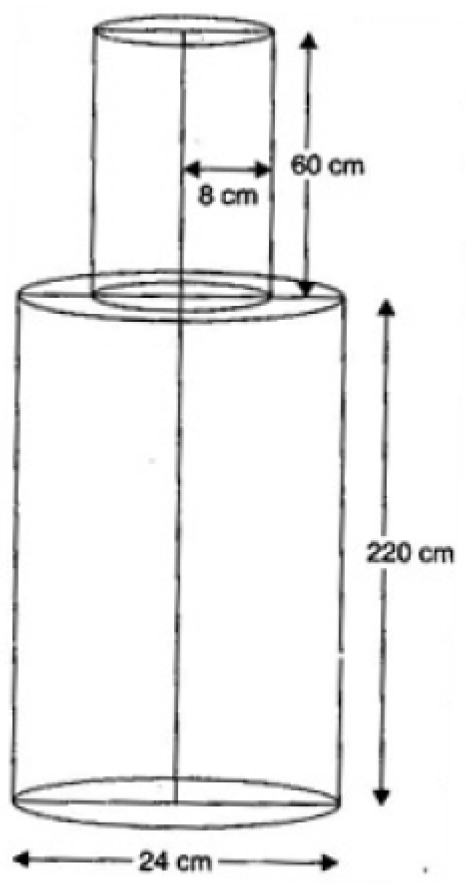
Ex 12.2 Question 6.

A solid iron pole consists of a cylinder of height $220 \mathrm{~cm}$ and base diameter $24 \mathrm{~cm}$, which is surmounted by another cylinder of height $60 \mathrm{~cm}$ and radius $8 \mathrm{~cm}$. Find the mass of the pole, given that $1 \mathrm{~cm}^3$ of iron has approximately $8 \mathrm{~g}$ mass. (Use $\pi = 3.14$)

Answer.

For lower cylinder, Base radius $(r) = \frac{24}{2} = 12 \mathrm{~cm}$
And Height $(h) = 220 \mathrm{~cm}$
$\text{Volume } = \pi r^2 h$
$= \pi(12)^2 \times 220$
$= 31680\pi \mathrm{cm}^3$

For upper cylinder, Base Radius $(R) = 8 \mathrm{~cm}$



And Height $(\mathrm{H}) = 60 \mathrm{~cm}$
$\text{Volume } = \pi \mathrm{R}^2 \mathrm{H}$
$= \pi(8)^2 \times 60$
$= 3840\pi \mathrm{cm}^3$
$\therefore$ Volume of the solid Iron pole
$=$ V of lower cylinder $+$ V of upper cylinder
$= 31680\pi + 3840\pi = 35520\pi$
$= 35520 \times 3.14 = 111532.8 \mathrm{~cm}^3$
$\therefore$ Volume of the solid Iron pole
$=$ V of lower cylinder $+$V of upper cylinder
mass of $1 \mathrm{~cm}^3$ iron $= 8\mathrm{gm}$
mass of $111532.8 \mathrm{~cm}^3$ iron $= 8^* 111532.8 = 892262.4\mathrm{gm} = 892.2624 \mathrm{~kg}$

Ex 12.2 Question 7.

A solid consisting of a right circular cone of height $120 \mathrm{~cm}$ and radius $60 \mathrm{~cm}$ standing on a hemisphere of radius $60 \mathrm{~cm}$ is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is $60 \mathrm{~cm}$ and its height is $180 \mathrm{~cm}$.

Answer.

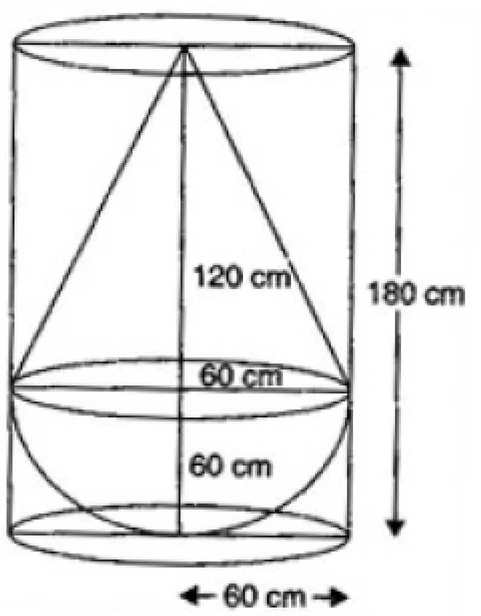
For right circular cone, Radius of the base $(r)=60 \mathrm{~cm}$
And Height $(h_1)=120 \mathrm{~cm}$

$$\text{Volume } =\frac{1}{3}\pi r^2 h_1$$
$$=\frac{1}{3}\pi(60)^2 \times 120$$
$$=144000\pi \mathrm{cm}^3$$

For Hemisphere, Radius of the base $(r)=60 \mathrm{~cm}$

$$\text{Volume } =\frac{2}{3}\pi r^3$$
$$=\frac{2}{3}\pi(60)^3$$
$$=144000\pi \mathrm{cm}^3$$

For right circular cylinder, Radius of the base $(r)=60 \mathrm{~cm}$
And Height $(h_2)=180 \mathrm{~cm}$



$$\text{Volume } =\pi r^2 h_2$$
$$=\pi(60)^2 \times 180$$
$$=648000\pi \mathrm{cm}^3$$

Now, V of water left in the cylinder

$$=\text{V of right circular cylinder } -(\text{V of right circular cone } + \text{V of hemisphere})$$
$$=648000\pi-(144000\pi+144000\pi)$$
$$=360000\pi \mathrm{cm}^3$$
$$=\frac{360000}{100 \times 100 \times 100}\pi \mathrm{m}^3$$
$$=0.36 \times \frac{22}{7}=1.131 \mathrm{~m}^3 \text{ (approx.)}$$

Ex 12.2 Question 8.

A spherical glass vessel has a cylindrical neck $8 \mathrm{~cm}$ long, $2 \mathrm{~cm}$ in diameter; the diameter of the spherical part is $8.5 \mathrm{~cm}$. By measuring the amount of water it holds, a child finds its volume to be $345 \mathrm{~cm}^3$. Check whether she is correct, taking the above as the inside measurements and $\pi=3.14$.

Answer.

For Cylinder: diameter of cylin. $=2 \mathrm{~cm}$, height of cylin. $=8 \mathrm{~cm}$
For Sphere : diameter of sphere $=8.5 \mathrm{~cm}$
Amount of water it holds $=\frac{4}{3}\pi r^3+\pi r^2 h=$ volume of sphere + volume of cylinder

$$=\frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3+\pi\left(\frac{2}{2}\right)^2 \times 8$$
$$=\frac{4}{3} \times 3.14 \times 4.25 \times 4.25 \times 4.25+8 \times 3.14$$
$$=321.39+25.12$$
$$=346.51 \mathrm{~cm}^3$$

Hence, she is not correct. The correct volume is $346.51 \mathrm{~cm}^3$.